JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 8, No. 4, Oct.–Dec. 1994

Transient Conjugate Heat Transfer Between a Laminar Stagnation Zone and a Solid Disk

Moh'd K. Alkam* and P. Barry Butler†
University of Iowa, Iowa City, Iowa 52242

An explicit finite difference technique is used to solve the case of transient, forced convective conjugate heat transfer between an axisymmetric incompressible laminar impinging jet and a solid disk. The solution to the governing equations is based on the assumptions of constant physical properties and no viscous dissipation. The hydrodynamics of the flow are considered to be steady state, whereas thermal transients take place with a step temperature change at the jet upstream boundary condition. The unexposed bottom surface of the disk is treated as an isothermal boundary, and the radial boundary of the disk is modeled as adiabatic. In this work, the effect of the ratios of thermal conductivity and thermal diffusivity between fluid and solid on transient heat transfer characteristics of the system have been investigated.

Nomenclature

H = nondimensional disk thickness, $h|V_0|/\alpha_f$
h = disk thickness
K_r = ratio between the thermal conductivity of the fluid and solid phases
L = nondimensional jet height, $l|V_0|/\alpha_f$
l = jet height
Q = nondimensional stagnation point heat flux, Q_w/Q_{ss}
Q_{ss} = stagnation point steady-state heat flux
Q_w = stagnation point heat flux
R = nondimensional radial coordinate, $r|V_0|/\alpha_f$
R_0 = nondimensional jet radius, $r_0|V_0|/\alpha_f$
r = radial coordinate
r_0 = jet radius
T = temperature
t = time
U = nondimensional axial velocity, $u/|V_0|$
u = axial velocity
V = nondimensional radial velocity, $v/|V_0|$
V_0 = jet velocity
v = radial velocity
Z = nondimensional axial coordinate, $z|V_0|/\alpha_f$
z = axial coordinate
α_r = ratio between the thermal diffusivity of the solid and that of the fluid
η = similarity variable
θ = nondimensional temperature, $(T - T_\infty)/(T_0 - T_\infty)$
ν = kinematic viscosity
τ = nondimensional time, tV_0^2/α_f
τ_{ss} = nondimensional time required to reach steady-state conditions
Ψ = stream function

Subscripts
f = fluid phase
s = solid phase
ss = steady state
0 = fluid jet
∞ = ambient

Received Oct. 14, 1993; revision received Jan. 10, 1994; accepted for publication Jan. 12, 1994.

*Graduate Research Assistant, Department of Mechanical Engineering.
†Associate Professor, Department of Mechanical Engineering. Member AIAA.

Introduction

TO date, a number of research articles treating transient, forced convective heat transfer have appeared in literature. The analysis and modeling of this process has received considerable attention since it has many engineering applications, including, the design of control systems for thermal devices, ignition of solid propellant by impingement of a hot gas generated by a pyrotechnic ignition device, and the treatment of thermal equipment operating under transient shutoff and startup conditions. A review of the literature shows that the geometry employed in many of the previous research efforts have been limited to axisymmetric channels, flat plates, and other generalized two-dimensional geometries. The following paragraphs summarize the relevant articles in this area of research, focusing primarily on theoretical studies that have investigated the problem using both analytical and numerical techniques.

Assuming uniform flow velocity (i.e., slug flow), Siegel[1] solved the case of transient forced convection in a parallel plate channel by an integral solution method. The case considered was that of unsteady heat transfer resulting from a stepwise wall temperature change. Dubrovich[2] also used an integral method to solve the same case under the same boundary conditions in a circular tube. Tsoi[3] implemented a Laplace transform technique to investigate the effect of an arbitrary cross section on the heat transfer characteristics of the transient flow in a circular pipe. This work was also performed with a step change in wall temperature. Perlmutter and Siegel[4] used an integral method to solve the case of laminar transient convective heat transfer between heated (or cooled) parallel plates. In their work, the unsteadiness is caused by simultaneously changing the fluid pumping pressure and either the wall temperature or the wall heat flux. The case was solved with the help of the "rod model." The rod model assumes that the fluid velocity is uniform along the channel section, but variable in time and equal to the mean flow velocity. Sparrow and Siegel[5] introduced the well-known Poiseuille velocity distribution to the energy equation to study the thermal entrance region of a circular tube under transient heating conditions caused by a step change in either wall temperature or wall heat flux. The case was also solved by means of an integral method. Siegel[6] also solved the same case under the same boundary conditions in a parallel plate channel.

Yang and Ou[7] investigated the effect of arbitrary time variant inlet velocity in the entrance region of both circular tubes and plane channels. The variant inlet velocity was introduced in Ref. 7 to generate transient forced convection. The cases of isothermal wall temperature and constant wall heat flux were considered. With the aid of the above mentioned rod model, and using an integral method, Siegel and Perlmutter[8] also studied the laminar transient convective heat transfer in a plane channel due to a variable heat flux with time and position. Delgado Domingos[9] solved the same problem in a circular tube.

Of special interest to the present study is the solution by Chao and Jeng[10] of impinging jet flow within the stagnation region. They obtained two asymptotic solutions, valid for small and large times, respectively. The case was solved for a step change in surface temperature and a step change in heat flux. Wang et al.[11] solved asymptotically both the exact energy equation and the boundary-layer energy equation for an axisymmetric impinging flow in the vicinity of the stagnation point with either nonuniform wall temperature or heat flux. An interesting study has been performed by Liu et al.[12] to investigate the stagnation-point heat transfer during impingement of laminar liquid jets. The study covered both the potential flow and the boundary-layer regions. Also, the authors included the effect of surface tension on the system, and they concluded that the surface tension has an effect on the stagnation-point flowfield when the Weber number is small. The aforementioned review shows briefly the main features of the analytical investigations which have been made in the field of transient, convective heat transfer. Clearly, there are many other contributors in this area of interest not cited here.

Advances in computer technology have made it possible to expand the types of problems considered in the analytical solutions, and treat rather complex cases using numerical solution algorithms. For example, Kettleborough[13] presented a numerical solution for the case of transient laminar free convection between two heated vertical plates, with constant wall temperature. The results of Ref. 12 included entrance effects and was solved using an explicit finite difference technique. In addition, the author assumed that the temperature gradients were small enough to neglect the density variation in the inertial terms (Boussinesq approximation). Yang et al.[14] also used an explicit finite difference method to investigate the effect of an oscillatory surface on the transient laminar natural convection on a vertical plate. El-Shaarawi and Alkam[15] solved the problem of transient forced convection in a concentric annulus with a step temperature change at one wall, and an adiabatic condition imposed on the second wall. The case was solved by an implicit finite difference technique.

Zumbrunnen[16] used the line-by-line method and a tridiagonal matrix algorithm to solve the finite difference equations resulting from the formulation of the case of steady-state convective heat and mass transfer in the stagnation region of a laminar planar jet impinging on a moving surface. In another study, Zumbrunnen[17] investigated the heat transfer characteristics of a transient planar stagnation flow against a stationary flat plate. Ramp and sinusoidal changes in the surface heat flux, or temperature have been applied. Results indicated that the time response is chiefly governed by the velocity gradient in the freestream, and to a lesser extent by Prandtl number. It should be emphasized that the numerical studies of transient, jet impingement heat transfer cited thus far have limited the solution of the energy equation to the fluid phase, with prescribed boundary conditions at the fluid-solid interface. Wang et al.,[18] however, investigated the analytical solution of the steady-state conjugate heat transfer between an impinging jet and a laterally insulated disk with arbitrary temperature or heat flux distribution prescribed on the unexposed bottom surface. Their solution of the liquid impingement problem is not limited to the stagnation region, but it is performed to cover the boundary-layer region. The results show that the heat transfer characteristics are dependent on Prandtl number of the fluid, the ratio of fluid thermal conductivity to solid thermal conductivity, the system dimensions, and the prescribed temperature or heat flux boundary conditions.

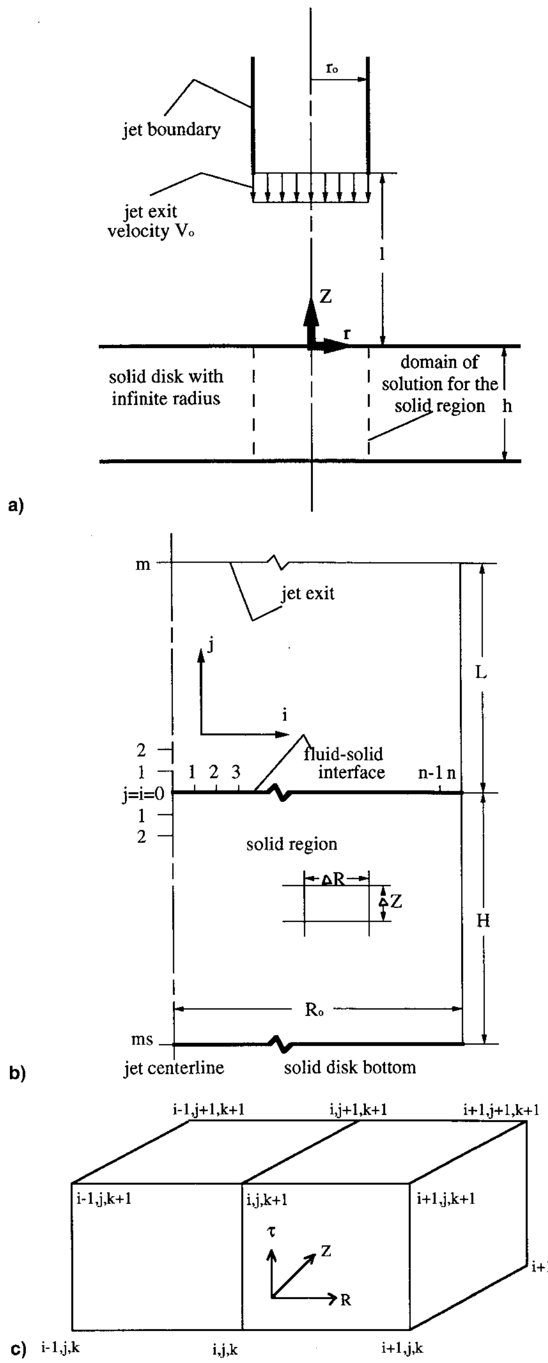

Fig. 1 a) Schematic diagram of the system, b) finite difference network in *R-Z* plane, and c) three-dimensional network with nondimensional time simulated as a third coordinate perpendicular to the *R-Z* plane.

In the present work, the unsteady conjugate heat transfer between an axisymmetric incompressible laminar jet and a solid disk is studied by means of an explicit finite difference technique. The solution is carried out for the stagnation region. To simulate the transient thermal response of a solid surface to a hot fluid jet, a temperature step at the jet outlet is imposed while the unexposed bottom surface of the solid disk is kept isothermal. Additional modeling assumptions imposed in the present solution include 1) steady-state hydrodynamics, 2) no viscous dissipation, and 3) constant physical properties. In the following sections, the development of the governing equations and a brief description of the numerical solution algorithm are presented. The resulting model is then applied to a typical impingement heating problem, and a study is made of the effect of thermal conductivity and thermal diffusivity ratios (i.e., interphase) on the transient heat transfer characteristics.

Governing Equations

Figure 1a depicts the geometry and coordinate system used in the problem under consideration. A parallel axisymmetric fluid jet with freestream velocity V_0, and jet radius r_0, impinges on the centerline of a solid disk located at $z = 0$. The jet is located a distance l above the surface of the solid disk, and the thickness of the solid disk is identified as h. Given the assumptions listed above, the well-known Hiemenz flow velocity distribution[19] (flow near a stagnation point) has been adopted to model the fluid phase. Hence, the velocity field in the fluid region is governed by the third-order, nonlinear ordinary differential equation, resulting from the well-known transformation of variables applied to Navier-Stokes equations.

In this analysis, the energy equations for both fluid and solid phases have been cast in terms of temperature under the assumptions of constant physical properties and no viscous dissipation. The energy equations are thus given in the standard dimensionless representation as

fluid-phase

$$\frac{\partial \theta_f}{\partial \tau} + U\frac{\partial \theta_f}{\partial Z} + V\frac{\partial \theta_f}{\partial R} = \frac{\partial^2 \theta_f}{\partial z^2} + \frac{1}{R}\frac{\partial}{R}\left(R\frac{\partial \theta_f}{\partial R}\right) \tag{1}$$

solid-phase

$$\frac{\partial \theta_s}{\partial \tau} = \alpha_r\left[\frac{\partial^2 \theta_s}{\partial z^2} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta_s}{\partial R}\right)\right] \tag{2}$$

All dimensionless variables appearing in Eqs. (1) and (2) are defined in the Nomenclature.

Initial and Boundary Conditions

The initial and boundary conditions imposed for this solution are intended to simulate the instantaneous introduction of a hot fluid on the top surface of a solid disk that is initially at certain reference temperature (T_0). Initially, the nondimensional temperature of the system is zero [$\theta(R, Z, \tau) = 0$], except at the jet outlet which is set to unity [$\theta(R, L, \tau) = 1$]. The bottom surface of the solid disk, which is unexposed to the fluid jet, is kept isothermal throughout the process [$\theta(R, -H, \tau) = 0$], and the disk is considered to be laterally insulated [$\partial\theta/\partial R(R_0, Z, \tau) = 0$]. Thus, heat loss of the disk is only allowed through the bottom isothermal surface. These imposed conditions in addition to the symmetry of the temperature distribution at the jet centerline, and the continuity of both temperature and heat flux at the interface, can be summarized as follows:

$$\theta_s(R, Z, 0) = \theta_f(R, Z, 0) = 0 \tag{3a}$$

$$\frac{\partial \theta_s}{\partial R}(0, Z, \tau) = \frac{\partial \theta_f}{\partial R}(0, Z, \tau) = 0 \tag{3b}$$

$$\frac{\partial \theta_s}{\partial R}(R_0, Z, \tau) = \frac{\partial \theta_f}{\partial R}(R_0, Z, \tau) = 0 \tag{3c}$$

$$\theta_f(R, L, \tau) = 1 \tag{3d}$$

$$\theta_s(R, -H, \tau) = 0 \tag{3e}$$

$$\frac{\partial \theta_f}{\partial Z}(R, 0^+, \tau) = \frac{1}{K_r}\frac{\partial \theta_s}{\partial Z}(R, 0^-, \tau) \tag{3f}$$

$$\theta_f(R, 0^+, \tau) = \theta_s(R, 0^-, \tau) \tag{3g}$$

Numerical Solution

The fluid velocity profiles were determined independent of the thermal fields by solving the governing equations of Hiemenz flow. This was done numerically using the multiple shooting technique for nonlinear, ordinary boundary value problems.[20] The velocity profiles have been validated by comparison with the results reported in Ref. 19. Once the radial and axial components of the velocity field were calculated, the fluid and solid phase transient energy equations were solved using a standard finite difference technique.

In the problem under consideration, there are three independent variables, namely, R, Z, and τ. Therefore, a three-dimensional parallelepiped grid was imposed on the solution domain. However, due to symmetry around the Z axis, the solution to only half of the domain was required. Figures 1b and 1c show the grid setup, in which τ is simulated as a third coordinate, normal to the R-Z physical plane. Mesh numbering has been carried out starting from an arbitrary origin, with i progressing in the radial direction ($i = 1$ at jet centerline), j progressing in the axial direction ($j = 1$ at fluid-solid interface), and k progressing in time ($k = 1$ at time $= 0$).

The energy equations were discretized explicitly, then applied to each grid node. Variables with subscript k are known from the previous time level, while those with subscript $k + 1$ are computed through a time marching process. This time marching process updates the temperature at the new time level ($k + 1$) as a function of the known values at the previous time level (k), using standard explicit finite differencing. Second-order central differencing has been applied to both R and Z derivatives, whereas time derivatives have been discretized in a standard first-order forward fashion. At the boundaries where Neumann boundary conditions are specified ($Z = 0$, $R = 0$, and $R = R_0$), a second-order, one-sided finite differencing has been used.

To assure high accuracy, several runs of the code have been carried out corresponding to different numbers of grid nodes. The results have been compared to detect any considerable changes attributed to the changes in step sizes. The final choice was 21 radial grid points, 31 axial grid points in each of the fluid and solid regions, and a nondimensional time step of $\Delta\tau = 0.1$.

To verify the numerical solution scheme, the program has been executed for a special case of a very large solid thickness ($H/L = 1000$). The results of this case compared very favorably with the known analytical solution of the steady-state heat conduction equation (linear profile).

Discussion of Results

The governing equations developed for this analysis describe 1) the (u, v) velocity field of the fluid component (Heimenz flow analysis), 2) the thermal field of the fluid phase, Eq. (1), and 3) the thermal field of the solid phase, Eq. (2). Together with the prescribed boundary conditions, Eqs. (3a–3g), the dimensionless system of equations requires the physical parameters α_r, K_r, and the H/L length ratio as input. For the first series of results presented here, values of $\alpha_r = 681.8$, $H/L = 10$, and $K_r = 0.0965$ are considered. These parameters correspond to the thermal properties of an air/coal interface. Based on the values selected for α_r, K_r, and H/L, the bottom of the disk is located at $Z = -127.8$, and the jet is located at $Z = 12.78$. To gain a better feeling of typical values of α_r and K_r, Table 1 is constructed for several different possible combinations of solids and fluids. The physical properties of the solids are taken at a temperature of 273 K, whereas those of the fluids are taken at 298 K. The reader should be aware

Table 1 Values of α_r and K_r for different solid-fluid combinations

Fluid/solid	α_r	K_r
Argon/copper	4.41×10^{-5}	5.55
Argon/glass wool	0.4658	7.9×10^{-3}
Water/copper	1.5162×10^{-3}	794.52
Air/coal	0.10833	6.33×10^{-3}

of the fact that the present results do not apply to a combustible solid at a temperature high enough to cause ignition.

Figures 2 and 3 present the nondimensional centerline ($R = 0$) temperature distributions for the fluid and solid regions, respectively. Unsteady thermal profiles at dimensionless times of $\tau = 5$, 20, 35, 45, and 100 are shown on each graph. For $\tau > 100$ the profiles are unchanged (i.e., steady state) with a nondimensional temperature of unity at the jet inlet ($Z = 12.78$), and a value of $\theta_g = \theta_s = 0.678$ at the fluid/solid interface ($Z = 0$). The in-depth thermal history of the solid disk (Fig. 3) is characteristic of the expected response of a finite thickness slab with time-varying heat flux at one boundary condition and an isothermal condition at the second boundary. Consistent with the classical heat conduction solution for a slab, the steady-state ($\tau > 100$) solution depicted in Fig. 3 shows that the predicted temperature distribution through the solid disk is linear, ranging from $\theta_s = 0.678$ at the gas interface ($Z = 0$) to $\theta_s = 0$ at the isothermal bottom surface ($Z = -127.8$).

A key parameter of interest in the present study is the transient heat flux imparted to the cold solid disk from the hot fluid jet. This is an important variable in assessing the thermal response of solid materials to an instantaneous blast of hot fluid in a direction normal to its surface. The nondimensional heat flux history Q (the heat flux normalized by its steady-state value) at the stagnation point ($R = 0$, $Z = 0$) for the baseline case is depicted in Fig. 4. The magnitude of Q, initially at zero, increases rapidly over $5 < \tau < 25$, and eventually approaches the steady-state value of unity at $\tau > 65$.

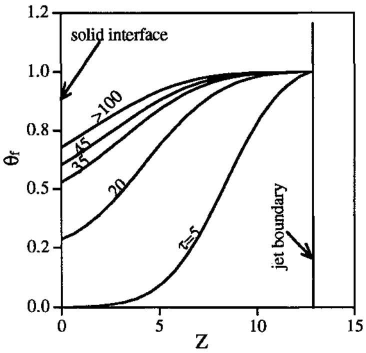

Fig. 2 History of nondimensional temperature distribution along jet centerline (fluid region) for $\alpha_r = 681.8$, $H/L = 10$, and $K_r = 0.0965$.

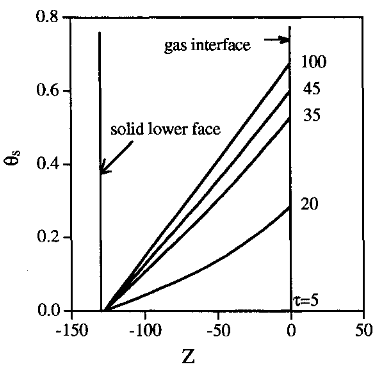

Fig. 3 History of nondimensional temperature distribution along jet centerline (solid region) for $\alpha_r = 681.8$, $H/L = 10$, and $K_r = 0.0965$.

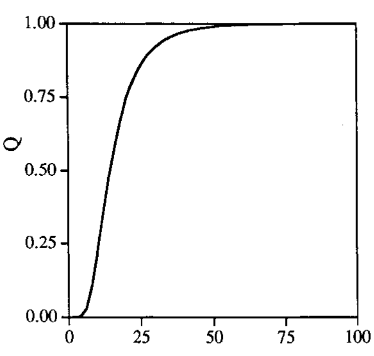

Fig. 4 Nondimensional stagnation point heat flux history for $\alpha_r = 681.8$, $H/L = 10$, and $K_r = 0.0965$.

As expected, one of the physical parameters which has a significant effect on the transient solution is the ratio of thermal conductivities of the fluid and solid, K_r. The effect of K_r on the nondimensional stagnation point heat flux is shown in Fig. 5. As the thermal conductivity of the solid decreases (K_r increases), it takes a longer time to reach steady-state operating conditions. In dimensionless Q-τ space, however, all three cases show a similar rate of rise in Q over the period $0 < \tau < 15$. Decreasing the solid thermal conductivity means effectively enhancing insulation effects on the fluid region, which will make the heat propagation less efficient across the interface, hence, it takes longer to reach steady state. For a better understanding of this behavior, Fig. 6 shows the effect of K_r on τ_{ss}. It should be noted that the steady-state condition is defined as the time when the residual reaches a predefined error value ($\varepsilon_r = 0.0001$). The residual is defined as the average of the absolute values of the temperature change through one time step all over the domain.

Figure 7 presents the effect of the thermal conductivity ratio on the stagnation point temperature history, demonstrating the relation between K_r and τ_{ss} (which was discussed earlier) and the steady-state magnitude of interface temperature $\theta_f(\tau_{ss}) = \theta_s(\tau_{ss})$.

Another important parameter which has a direct impact on the heat transfer characteristics of transient impingement heating, is the ratio between the thermal diffusivity of the solid and that of the fluid α_r. Figure 8 shows the effect of α_r on the time required for the system to reach steady state. Increasing the value of α_r (holding K_r constant) implicitly means decreasing the heat capacity of the solid. As such, a

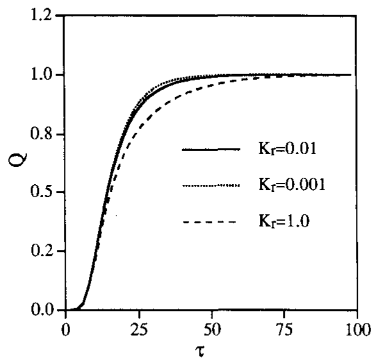

Fig. 5 Effect of thermal conductivity ratio on nondimensional stagnation point heat flux.

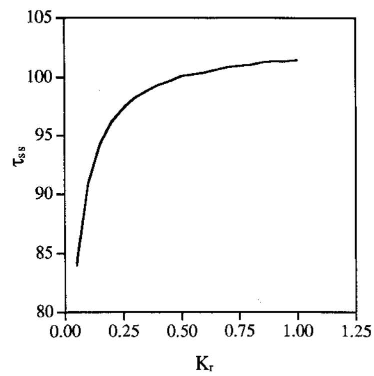

Fig. 6 Effect of thermal conductivity ratio on time required to reach steady-state thermal field.

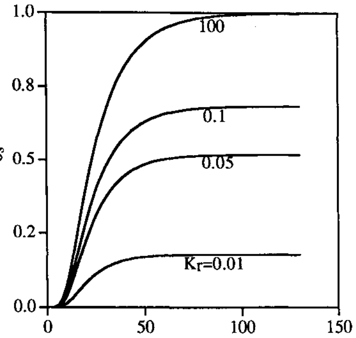

Fig. 7 Effect of thermal conductivity ratio on nondimensional stagnation temperature history.

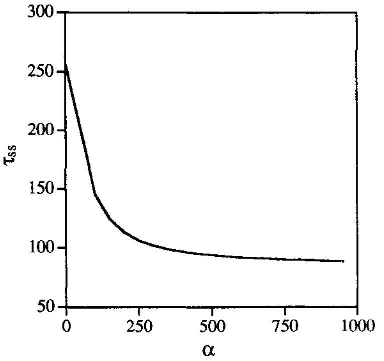

Fig. 8 Effect of thermal diffusivity ratio on time required to reach steady-state thermal field.

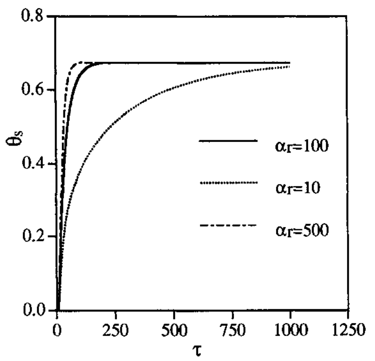

Fig. 9 Effect of thermal diffusivity ratio on nondimensional stagnation temperature history.

lower heat capacity results in a faster convergence to steady-state conditions. Figure 9 presents the effect of α_r on stagnation point temperature history for three values of α_r. As expected, as α_r increases the system reaches steady state faster due to the decrease in solid heat capacity.

The effect of the H/L ratio on the stagnation region nondimensional heat flux history is exhibited in Fig. 10. To understand the effect of the H/L ratio on the system under consideration, it is instructive to examine the two extreme situations corresponding to very small and very large H/L values. As H/L approaches zero, the simulated physical problem will be of the impingement of a fluid against a zero thickness sheet. In this case the nondimensional stagnation region heat flux is expected to grow from zero initially to unity at steady-state conditions. On the other hand, as H/L approaches a very large value ($\gg 1$), the physical phenomenon is interpreted as a step temperature change at the top of a solid disk. In this extreme case, the nondimensional stagnation region heat flux is maximum at the beginning of the process, then decreases gradually until it reaches unity at steady-state conditions. Figure 10 shows the behavior of the nondimensional heat flux for several values of H/L ratio (10, 20, 30, 40)

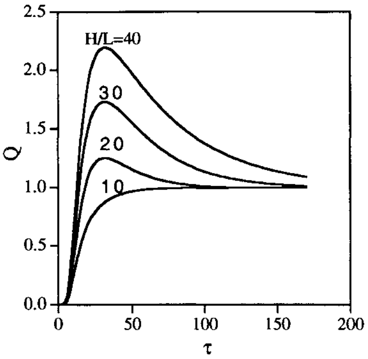

Fig. 10 Effect of H/L ratio on the stagnation point heat flux history.

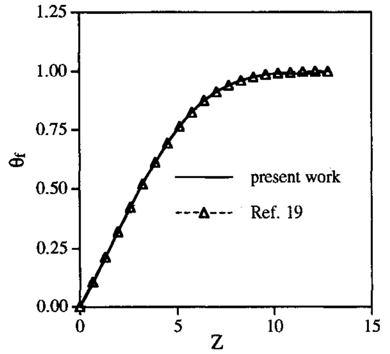

Fig. 11 Steady-state temperature profile in the fluid region, with H/L = 1.0, and K_r = 0.00001 (comparison with White[19]).

between the two previously mentioned limits. It is clear that for low values of H/L, the behavior of the former extreme case dominates, while for large values of H/L, the profiles tend to have the trend of the latter extreme case.

It is worth mentioning that Reynolds and Prandtl numbers do not appear explicitly in the governing equations; these parameters have been absorbed into the dimensionless variables. This was done to carry out a compact presentation of the results. As in any forced convection heat transfer problem, Prandtl and Reynolds numbers do influence the heat transfer characteristics of the system. However, in the current presentation, the effect of those familiar parameters is not exhibited explicitly. Another fact to mention is that the study of the stagnation region cannot detect any radial change in the Nusselt number in the impingement problems. However, to do so, one must extend the domain of the solution to adjacent regions (boundary layer and/or similarity regions) as was done in the time-independent study by Wang et al.[16]

To validate the present results, the code has been executed for a limiting case where the thermal conductivity of the solid is extremely high ($K_r = 10^{-5}$). In theory, the steady-state temperature profile within the fluid region predicted by the present analysis should be comparable to the well known stagnation region temperature distribution given by White.[19] The comparison depicted in Fig. 11 shows good agreement between Ref. 19 and the present analysis for this limiting case.

Conclusions

In this work an explicit finite difference scheme has been used to solve the problem of transient impingement heating. The fluid was treated as an incompressible and laminar flow with constant physical properties. Results for typical impingement cases have been presented to identify the main heat transfer characteristics of the problem under consideration. Among these results are histories of temperature profiles, stagnation point heat flux, and the effect of some important parameters (i.e., thermal conductivity ratio and thermal diffusivity ratio) on heat transfer characteristics.

References

[1]Siegel, R., "Heat Transfer for Laminar Flow in Ducts with Arbitrary Time Variation in Wall Temperature," *Transactions of the American Society of Mechanical Engineers, Series E, Journal of Applied Mechanics*, Vol. 52, 1960, pp. 241–249.

[2]Dubrovich, E. M., "Transient Heat Transfer for Laminar Fluid Flow in Tubes," Teoriya, Kkonstr. Rascheti. Ispyt, Dvigat Vuytr. Sgoraniya, (Trudy Inst. Dvigatelei AN SSSR), Vol. 6, Izd. Akad, Nauk SSSR, Moscow, 1962, pp. 51–60.

[3]Tsoi, P. V., "One Method of Studying Transient Heat Transfer for a Forced Fluid Flow in Tubes of Arbitrary Section," *Teplo-i massopersons*, Vol. 1, No. 8, Izd. Energia, Moscow, 1968, pp. 57–61.

[4]Perlmutter, M., and Siegel, R., "Unsteady Laminar Flow in a Duct with Unsteady Heat Addition," *Journal of Heat Transfer*, Vol. 83, 1961, pp. 432–440.

[5]Sparrow, E. M., and Siegel, R., "Thermal Entrance Region of a Circular Tube Under Transient Heating Conditions," *Proceedings of the Third U.S. Congress for Applied Mechanics*, 1958, pp. 817–826.

[6]Siegel, R., "Transient Heat Transfer for Laminar Slug Flow in Ducts," *Journal of Applied Mechanics*, Vol. 26, No. 1, 1959, pp. 140–142.

[7]Yang, W. J., and Ou, J. W., "Unsteady Forced Convection of the Entrance Region of Closed Conduits Due to Arbitrary Time Variant Inlet Velocity," *ASME Semi-International Symposium Heat Mass Transfer*, Vol. 1, Tokyo, 1967, pp. 133–143.

[8]Siegel, R., and Perlmutter, M., "Laminar Heat Transfer in a Channel with Unsteady Flow and Wall Heating Varying with Position and Time," *Journal of Heat Transfer*, Vol. 85, 1963, pp. 358–365.

[9]Delgado Domingos, J., "Transmissao do Calor num Tubo em Escoamento Laminar e Regime nao Estacianario," Vol. 40, No. 352, 1965, pp. 65–74.

[10]Chao, B. T., and Jeng, D. R., "Unsteady Stagnation Point Heat Transfer," *Journal of Heat Transfer*, 1965, p. 221.

[11]Wang, X. S., Dagan, Z., and Jiji, L. M., "Heat Transfer Between a Circular Free Impinging Liquid Jet and a Solid Surface with Non-Uniform Wall Temperature or Wall Heat Flux-1. Solution of the Stagnation Region," *International Journal of Heat and Mass Transfer*, Vol. 32. No. 7, 1989, pp. 1351–1360.

[12]Liu, X., Gabour, L. A., and Lienhard, J. H., "Stagnation-Point Heat Transfer During Impingement of Laminar Liquid Jets: Analysis Including Surface Tension," *Journal of Heat Transfer*, Vol. 115, No. 1, 1993, pp. 99–105.

[13]Kettleborough, C. F., "Transient Laminar Free Convection Between Heated Vertical Plates Including Entrance Effects," *International Journal of Heat and Mass Transfer*, Vol. 15, No. 5, 1972, pp. 883–896.

[14]Yang, J. W., Caccia, C., and Goodman, J., "Laminar Natural Convection About Vertical Plates with Oscillatory Surface Temperature," *Journal of Heat Transfer*, Vol. 96, No. 1, 1974, pp. 9–14.

[15]El-Shaarawi, M. A. I., and Alkam, M. K., "Transient Forced Convection in the Entrance Region of Concentric Annuli," *International Journal of Heat and Mass Transfer*, Vol. 35, No. 12, 1992, pp. 3335–3344.

[16]Zumbrunen, D. A., "Convective Heat and Mass Transfer in the Stagnation Region of a Laminar Jet Impinging on a Moving Surface," *American Society of Mechanical Engineers Transactions, Journal of Heat Transfer*, Vol. 113, No. 3, 1991, pp. 563–570.

[17]Zumbrunen, D. A., "Transient Convective Heat Transfer in Planar Stagnation Flows with Time-Varying Surface Heat Flux and Temperature," *Journal of Heat Transfer*, Vol. 114, 1992, pp. 85–93.

[18]Wang, X. S., Dagan, Z., and Jiji, L. M., "Conjugate Heat Transfer Between a Laminar Impinging Liquid Jet and a Solid Disk," *International Journal of Heat and Mass Transfer*, Vol. 32, No. 11, 1989, pp. 2189–2197.

[19]White, F. M., *Viscous Fluid Flow*, McGraw-Hill, New York, 1974.

[20]Subroutine BVPMS, IMSL Math Library, Univ. of Iowa ICAEN computing network, Iowa City, IA.